Chiral Symmetry Breaking in the 3-d Thirring model for small N_f.

L. Del Debbio [a]* (for the UKQCD Collaboration)

[a]Centre de Physique Théorique, CNRS Luminy, F-13288 Marseille Cedex 9, France

We study the dynamical breaking of chiral symmetry in the 3-d Thirring model for a small number of fermion species. The critical point is identified by fitting lattice data to an equation of state. The spectrum of the theory is studied to confirm the phase structure of the model.

1. Introduction

The Thirring model is a three dimensional QFT with N_f fermion species and a current–current four fermion interaction. As such, it is obviously non–renormalisable in the usual perturbative expansion. It has been pointed out several years ago [1] that a continuum limit can be defined for some theories in a non–perturbative (NP) approach. For the Gross–Neveu model, a four–fermi theory with a (pseudo–)scalar interaction, the existence of such an UV fixed point has been shown within the $1/N_f$ expansion and confirmed by numerical simulations. The critical point separates a strong coupling phase with chiral symmetry breaking from a symmetric weak–coupling one.

For the Thirring model, solid conclusions can not be drawn from analytic results as different NP techniques yield different results. We aim to obtain a complete description of the phase structure of this model by lattice simulation, using staggered fermions and a HMC algorithm. The motivations, the lattice formulation of the theory and the technical details of our analysis were presented in great details in earlier works [2], to which we refer the interested reader. This contribution summarizes our most recent results for $N_f = 4, 6$.

The order parameter for detecting chiral symmetry breaking is the chiral condensate $\langle \bar{\chi}\chi \rangle$. Its behaviour around the critical point is described by fitting to an equation of state (EOS) [3]. Finite–size effects are included in the EOS and the critical exponents are obtained for $N_f = 4$.

The results are compared with those for $N_f = 2$. Once again, lattice data show a clear signal of chiral symmetry breaking and fit rather well the RG predictions.

Moreover the chiral symmetry breaking transition should manifest itself in the behaviour of the susceptibilities and in the spectrum of the theory. This issue was already addressed for $N_f = 2$ in [2]. Preliminary results for $N_f = 4$ are reported here. The general pattern, which had already emerged in our previous works, is confirmed by the new results. We briefly outline the next developments in our program.

2. Fitting to the Equation of State

The EOS is a functional relation between the order parameter, the coupling, g, and the bare mass, m, in the regularised theory, which can be derived form the RG equations in the vicinity of a fixed point (see e.g. [3]). It therefore provides a relation to which lattice data can be fitted. The results of this approach for $N_f = 2$ and fixed lattice size were presented in [2]. In order to take into account finite size effects, it is sufficient to consider the inverse linear size of the lattice, $1/L$ as an additional scaling field of dimension 1. The exact functional form of the EOS is unknown, but it can be expanded in Taylor series, yielding a fitting function for the lattice data with only a few unknown parameters:

$$m = B\langle \bar{\chi}\chi \rangle^{\delta} + A(t + CL^{-1/\nu})\langle \bar{\chi}\chi \rangle^{\delta-1/\beta} \qquad (1)$$

where $t = 1/g^2 - 1/g_c^2$.
For $N_f = 4$, we fit to the EOS data from lattice whose sizes range from 8^4 to 16^4. The results are

---

Table 1
Results from fits including finite size scaling.

	Parameter	Fit		Parameter	Fit
$N_f = 2$	$1/g_c^2$	1.92(2)	$N_f = 4$	$1/g_c^2$	0.70(2)
	δ	2.75(9)		δ	3.41(31)
	β	0.57(2)		β	0.41(10)
	η	0.60(2)		η	0.34(20)
	ν	0.71(4)		ν	0.61(10)
	A	0.334(7)		A	0.70(5)
	B	2.7(3)		B	5.8(2.1)
	C	2.1(7)		C	2.9(2.2)
	χ^2/d.o.f	1.76		χ^2/d.o.f	0.80

summarized in Tab. 1.

It is interesting to stress that the results from the fit do take into account the effects due to the finite volume of the lattice and should therefore be interpreted as values in the thermodymamical limit. The values for the critical coupling and the critical exponents are stable with respect to the previous results obtained for $N_f = 4$ at fixed lattice size [2]. Such a stability in our fits can be seen as a sign that the lattice data really follow the behaviour expected in the vicinity of an UV fixed point. The critical exponents differ significantly between $N_f = 2$ and $N_f = 4$, confirming a result that had already emerged from the fixed size fits. We should be able to reduce the errors on the fitted values as statistics increase over the next months. It is worthwhile to stress once again that, in order to identify unambiguously a fixed point, we should study the renormalised trajectories in coupling space, for which we have to measure 3- and 4-point correlation functions.

3. Susceptibilities and spectrum

Independent information on the phase structure of the theory can be obtained from the susceptibilities and the spectrum of the theory. Once again precise definitions and the results for the $N_f = 2$ case are given in previous works and we want to concentrate on the new results for $N_f = 4, 6$. We briefly recall the definitions and the expected behaviour for some quantities of interest.

The transverse and longitudinal susceptibilities are defined respectively as:

$$\chi_l = \frac{1}{V} \sum_x \langle \bar{\chi}\chi(0) \, \bar{\chi}\chi(x) \rangle_c \tag{2}$$

$$\chi_t = \frac{1}{V} \sum_x \langle \bar{\chi}\varepsilon\chi(0) \, \bar{\chi}\varepsilon\chi(x) \rangle_c = \frac{1}{m} \langle \bar{\chi}\chi \rangle \tag{3}$$

They are related to the scalar and pion masses:

$$\chi_{l,t} = Z_{s,\pi}/M_{s,\pi}^2 \tag{4}$$

Another interesting quantity is their ratio $R = \chi_l/\chi_t$. If there is a critical point in the phase diagram where chiral symmetry is dynamically broken, then:

$$\lim_{m \to 0} R = \begin{cases} 0, & g^2 > g_c^2; \\ \frac{1}{\delta - \frac{1}{\beta}}, & g^2 < g_c^2. \end{cases} \tag{5}$$

The results for the susceptibilities are shown in Fig. 1. Even though the results were produced at non-vanishing bare mass, i.e. in presence of an explicit symmetry breaking term, the pattern reflects the existence of a phase transition. Once again the near degeneracy of χ_l and χ_t in the symmetric phase ($t > 0$) supports the hypothesis $\delta - 1/\beta = 1$, used in the fits reported in Tab. 1.

In the strong coupling regime ($t < 0$), the transverse susceptibility is larger than the longitudinal, reflecting the fact that the pion is almost a Goldstone boson in that regime. The same trend is confirmed by the masses extracted from the aymptotic behaviour of two–point functions. Preliminary results are reported in Fig. 2; the errors on the scalar mass is still very large and has not been included in the plot. A more detailed
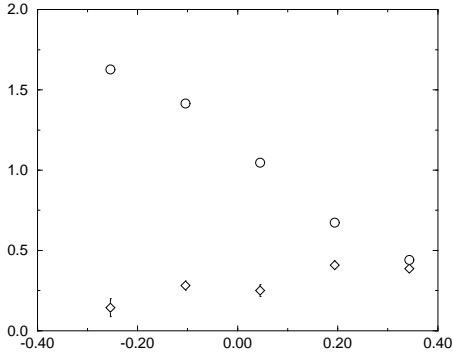
Figure 1. Transverse (circles) and longitudinal (diamonds) susceptibilities for $N_f = 4$, $m = 0.01$, on a 16^3 lattice. The data are plotted against a rescaled reduced coupling $t = g_c^2(1/g^2 - 1/g_c^2)$.

analysis, based on larger statistical samples, is currently in progress and we hope to report soon on the details of the spectrum for $N_f = 4$. It is nonetheless worth to remark that the general pattern is consistent with the existence of an UV fixed point. In particular, we see no sign for the unconventional spectrum in the symmetric phase associated to conformal phase transitions, as suggested in [4].
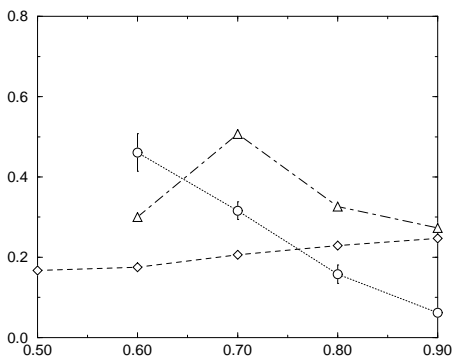

Figure 2. Fermion (circles), pion (diamonds) and scalar (triangles) masses vs. $1/g^2$, for $N_f = 4$, $m = 0.01$, on a 16^3 lattice.

In order to distinguish between different scenarios of chiral symmetry breaking (corresponding, e.g., to different solutions of the Schwinger–Dyson equations), several values of N_f need to be studied. New preliminary results for the chiral condensate are reported in Fig. 3. The lattice used is larger than in our earlier works [2] and the behaviour of the condensate as the bare mass m is decreased suggests the existence of a symmetry broken phase at strong coupling. We will report more systematic results in the near future.
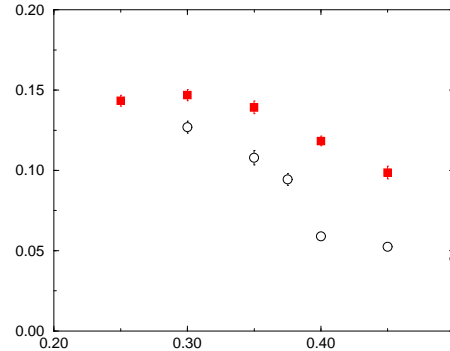

Figure 3. Chiral condensate vs. $1/g^2$ for $N_f = 6$, $m = 0.01$ (circles) on a 16^3 lattice and $m = 0.02$ (squares) on a 12^3 lattice.

REFERENCES

1. G. Parisi, *Nucl. Phys.* **B 100** (1975) 368.
2. L. Del Debbio, S.J. Hands, *Phys. Lett.* **B 373** (1996) 171; L. Del Debbio, *Nucl. Phys.* **B** (Proc. Suppl.) **53** (1997) 699; L. Del Debbio, J.C. Mehegan, S.J. Hands, hep-lat/9701016, accepted for publication in *Nucl. Phys. B.*
3. J. Zinn-Justin, "Quantum Field Theory and Critical Phenomena", second edition, Clarendon Press, Oxford 1993.
4. V.A. Miranskii, K. Yamawaki, *Phys. Rev.* **D 55** 5051 (1997).